A recursive formula for Thabit numbers

Zenodo 10.5281/zenodo.2638790

A recursive formula for Thabit numbers

Amelia Carolina Sparavigna

Politecnico di Torino

Abstract Here we discuss the Thabit numbers. An operation of addition of these numbers is proposed. A recursive relation is given accordingly.

Keywords Thabit numbers

DOI: 10.5281/zenodo.2638790

Written in Turin, on April 13, 2019

In recent papers we have discussed some properties of the Mersenne numbers [1,2] and of the Fermat numbers [3], using an approach based on generalized operations of addition [4-8]. In [9], we have discussed the Cullen and Woodall numbers too (for references on these numbers, see [10-13]). Here we consider the Thabit numbers [14]. These numbers are given as $T_n=3*2^n-1$, where the asterisk represents the ordinary multiplication.

Let us consider the following operation:

$$T_{m+n}=T_m\oplus T_n$$

Therefore

$$T_{m+n}=3*2^{m+n}-1=3*2^{m+n}-1+2^n-2^n=2^n(3*2^m-1)-1+2^n=2^n T_m+2^n-1$$

$$T_{m+n}=2^n\frac{3}{3}T_m+2^n-1-\frac{T_m}{3}+\frac{T_m}{3}=\frac{1}{3}T_m T_n+\frac{3}{3}2^n-\frac{1}{3}+\frac{1}{3}-1+\frac{T_m}{3}$$

So we have:

$$(1)\quad T_m\oplus T_n=\frac{1}{3}T_m T_n+\frac{1}{3}T_m+\frac{1}{3}T_n-\frac{2}{3}=\frac{1}{3}(T_m+T_n+T_m T_n-2)$$

Using (1), we can see that the neutral element is $T_0=2$, so that:

$$T_m\oplus T_0=\frac{1}{3}(T_m+T_0+T_m T_0-2)=\frac{1}{3}(3T_m)=T_m$$

The recursive relation is given accordingly to (1), starting from $T_1 = 5$:

$$T_{n+1} = T_n \oplus T_1 = \frac{1}{3}(T_n + T_1 + T_n T_1 - 2) = \frac{1}{3}(T_n + 5 + 5\,T_n - 2) = \frac{1}{3}(6\,T_n + 3) = 2\,T_n + 1$$

With a Fortran program (double precision), we have **5**, **11**, **23**, **47**, 95, **191**, **383**, 767, 1535, 3071, **6143**, 12287, 24575, 49151, 98303, 196607, 393215, **786431**, 1572863, 3145727, 6291455, 12582911, 25165823, 50331647, 100663295, 201326591, 402653183, 805306367, 1610612735, 3221225471, 6442450943, 12884901887, 25769803775, **51539607551**, 103079215103, 206158430207, 412316860415, **824633720831**, 1649267441663, 3298534883327, 6597069766655, 13194139533311, **26388279066623**, 52776558133247, 105553116266495, 211106232532991, 422212465065983, 844424930131967, 1688849860263935, 3377699720527871, 6755399441055743. In bold characters, the prime numbers as from http://oeis.org/A007505.

References

[1] Sparavigna, A. C. (2018). The q-integers and the Mersenne numbers. Zenodo. http://doi.org/10.5281/zenodo.1251833

[2] Sparavigna, A. C. (2018). On a generalized sum of the Mersenne Numbers. Zenodo. http://doi.org/10.5281/zenodo.1250048

[3] Sparavigna, A. C. (2018). The group of the Fermat Numbers. Zenodo. http://doi.org/10.5281/zenodo.1252422

[4] Sparavigna, A. C. (2018). Generalized Sums Based on Transcendental Functions. Zenodo . DOI: 10.5281/zenodo.1250020

[5] Sparavigna, A. C. (2018). On the generalized sum of the symmetric q-integers. Zenodo. DOI: 10.5281/zenodo.1248959

[6] Sparavigna, A. C. (2018). ON THE ADDITIVE GROUP OF q-INTEGERS. Zenodo . DOI: 10.5281/zenodo.1245849

[7] Sicuro, G., & Tempesta, P. (2016). Groups, information theory, and Einstein's likelihood principle. Phys. Rev. E 93, 040101(R).

[8] Curado, E. M., Tempesta, P., & Tsallis, C. (2016). A new entropy based on a group-theoretical structure. Annals of Physics, 366, 22-31.

[9] Sparavigna, A. C. (2019). On the generalized sums of Mersenne, Fermat, Cullen and Woodall Numbers. Zenodo. http://doi.org/10.5281/zenodo.2634312

[10] Weisstein, Eric W. "Mersenne Number." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/MersenneNumber.html

[11] Weisstein, Eric W. "Fermat Number." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/FermatNumber.html

[12] Weisstein, Eric W. "Cullen Number." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/CullenNumber.html

[13] Weisstein, Eric W. "Woodall Number." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/WoodallNumber.html

[14] Weisstein, Eric W. "Thâbit ibn Kurrah Number." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/ThabitibnKurrahNumber.html